A Multiple Step-like Spectrum of Primordial Perturbation

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Abstract

We show that if the inflaton effective potential has multiple discontinuous points in its first derivative, the spectrum of primordial perturbation will be multiple step-like. We give a general analysis by applying a simple model. In principle, as long as the height of step is low enough, the result of spectrum will be consistent with observations.

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I. INTRODUCTION

Inflation[1],[2],[3] is a stage of an exponential expansion, which brings the current observable universe. During the period of inflation, the quantum fluctuations inside the horizon would be able to extend out the horizon and become the primordial perturbations responsible for the structure formation of observable universe, see Ref.[4]. The primary advantages of inflation and the accumulating observational data make the inflation scenario become a leading candidate of the primordial universe.

The primordial perturbations obtained during inflation is approximately scale invariant and Gaussian, all of which have been confirmed by experiments to some extent. In some inflationary models, inflaton potentials has a step, and the steplike change in the potential will result in an universal oscillation in the spectrum of primordial perturbations [5],[6],[7],[8],[9],[10],[11],[12],[13],[14],[15],[16],[17],[18],[19]. In other inflationary models, the oscillation of primordial spectrum can be also generated, e.g. inflaton potential with a small oscillation [20],[21],[22],[23], the field decay during inflation [24], or the particles production during inflation [25],[26],[27], the bounce before inflation [28],[29],[30], the heterotic M-theory [31]. Interestingly, a burst of oscillations in the primordial spectrum seems to provide a better fit to the CMB angular power spectrum[32, 33]. However, given the improvements in data quality anticipated with coming observations, it is significant to ask whether arbitrary features in the primordial power spectrum can be acquired during inflation. Therefore, it is worth to search other possibilities of the features of the primordial power spectrum.

As has been pointed in [34],[35],[36], if the inflationary potential has a discontinuity in its first derivative at certain point, there will a step-like behavior in its power spectrum. As long as the height of this step is low enough, the inflationary power spectrum obtained can be still agreement with the observations. Recently, the inflationary models based on the string landscape have been intensively studied. It might be possible that there are multiple discontinuous points in the first derivative of inflationary effective potential. In this paper, we analytically and numerically show that this might imply a multiple step-like spectrum of primordial perturbation. We will give a general analysis by applying a simple model, and therefore the conclusion is somewhat universal.

The plan of this work is as follows. The power spectrum of perturbations is calculated in section II. The numerical results of this power spectrum are showed in section III. In section

IV, we discuss the applications of results.

II. THE CALCULATIONS OF POWER SPECTRUM

We begin with the inflaton effective potential with a discontinuity in its first derivative at some points. Hereafter, we call such points as the discontinuous points for convenience. In the paper, we only discuss the simplest case of the effective potential, i.e. the linear potential except the discontinuous point. We define $V_{,\varphi} = d_j$ for $\varphi_j < \varphi < \varphi_{j-1}$, where d_j is the slope of the potential in the corresponding region and is constant, and $d_j > 0$, $d_j \neq d_{j-1}$. In each region, $V_{,\varphi\varphi} = 0$, thus the slow roll conditions can be satisfied for small d_j. However, at the discontinuous points (φ_j),

$$V_{,\varphi\varphi} = \sum_{j=1}^{n} \mathcal{D}_{j-1,j}\, \delta(\varphi - \varphi_j), \tag{1}$$

where $\mathcal{D}_{j-1,j} = d_{j-1} - d_j$, and n is the number of the discontinuous points. The sum of delta functions is rewritten as

$$V_{,\varphi\varphi} \simeq \frac{3H}{a} \sum_{j=1}^{n} \frac{\mathcal{D}_{j-1,j}}{d_j}\, \delta(\eta - \eta_j), \tag{2}$$

where $3H\dot{\varphi} \simeq -V_{,\varphi}$ and Eq.(1) are applied. Here, the delta functions are obviously induced by the discontinuity of first derivative of the effective potential, which will inevitably appear in the perturbation equation via $V_{,\varphi\varphi}$. We will check the effect of such discontinuous points on the primordial perturbation in the following.

The equation of motion for the scalar perturbation $\delta\varphi$ during inflation in k space is given by, after defining $v = a\delta\varphi$, and $x = k\eta$,

$$\frac{d^2 v}{dx^2} + [1 - \frac{2}{x^2} + \sum_{j=1}^{n} \frac{\sigma_j}{x} \delta(x - x_j)]v = 0, \tag{3}$$

where $\sigma_j = \frac{3\mathcal{D}_{j-1,j}}{d_j} \ll 1$ is required by the observations, and $2/x^2$ is given by $a''/a \simeq 2/\eta^2$. In principle, although the field, due to some mechanisms, suddenly experiences a change of its slope, the change should be not large. σ_j is critical for the shape of the power spectrum, which will be found.

The solutions of Eq.(3) are,

$$v_j = a_j(i + \frac{1}{x})e^{-ix} + b_j(-i + \frac{1}{x})e^{ix}, \tag{4}$$

where a_j and b_j are dependent of k. In general, initially the perturbation mode is deep inside the horizon, which implies $a_0 \sim \frac{1}{\sqrt{2k}}, b_0 = 0$, i.e.

$$v_0 = a_0(i + \frac{1}{x})e^{-ix}. \tag{5}$$

By applying the junction condition

$$v_{j-1}(x_j) = v_j(x_j),\ [\frac{dv}{dx}]_{x_j} = -\frac{v(x_j)}{x_j}\sigma_j, \tag{6}$$

we have the Bogoliubov coefficients

$$\begin{aligned} a_j &= [1 + \frac{\sigma_j}{2ix_j}(1 + \frac{1}{x_j^2})]a_{j-1} + \frac{\sigma_j}{2ix_j}(\frac{1}{x_j} - i)^2 e^{2ix_j} b_{j-1}, \\ b_j &= -\frac{\sigma_j}{2ix_j}(i + \frac{1}{x_j})^2 e^{-2ix_j} a_{j-1} + [1 - \frac{\sigma_j}{2ix_j}(1 + \frac{1}{x_j^2})]b_{j-1}, \end{aligned} \tag{7}$$

where $x_j = k\eta_j$. This is a set of recursive equations, by which a_j and b_j can be related to a_0 and b_0. We are interested in the perturbations on large scale, i.e. evaluated at $x = k\eta \to 0$. Thus from Eq.(4), we have

$$v = a_n(i + \frac{1}{x})e^{-ix} + b_n(-i + \frac{1}{x})e^{ix} \simeq \frac{a_n + b_n}{x} \ \ for\ \ x \to 0. \tag{8}$$

We get

$$|v|^2 = \frac{1}{|x|^2}[|M_n|^2(|a_{n-1}|^2 + |b_{n-1}|^2) + 2Re(M_n^2 a_{n-1} b_{n-1}^*)], \tag{9}$$

$$M_n = 1 + \frac{\sigma_n}{x_n}(\frac{sinx_n}{x_n} - cosx_n)(\frac{1}{x_n} + i)e^{-ix_n}. \tag{10}$$

Therefore the power spectrum of $\delta\varphi$ is

$$\mathcal{P}_{\delta\varphi}(k) = \frac{k^3}{2\pi^2}|\frac{v}{a}|^2, \tag{11}$$

and the spectral index is

$$n_{\delta\varphi} - 1 = \frac{d\ln\mathcal{P}_{\delta\varphi}(k)}{d\ln k}. \tag{12}$$

where v_k is given by Eq.(9), which is a set of recursive equations and is difficultly to be solved for the case $n \gg 1$. Thus we will give the numerical results of spectrum in following section.

III. THE NUMERICAL RESULTS

When the effective potential of field only includes a discontinuous point, for Eqs.(7), we read

$$\begin{aligned} a_1 &= a_0[1+\frac{\sigma_1}{2ix_1}(1+\frac{1}{x_1^2})], \\ b_1 &= -a_0\frac{\sigma_1}{2ix_1}(i+\frac{1}{x_1})^2 e^{-2ix_1}. \end{aligned} \tag{13}$$

Thus with Eq.(8), we have

$$v \approx \frac{a_0}{x}[1+\frac{\sigma_1}{x_1}(\frac{sinx_1}{x_1}-cosx_1)(\frac{1}{x_1}+i)e^{-ix_1}]. \tag{14}$$

The result is that obtained in Ref.[32]. The power spectrum generally has a jump, which is leaded by the discontinuous point, see also [37].

In this paper, we will see that for an effective potential with multiple discontinuous points, the power spectrum will has a multiple step-like behavior.

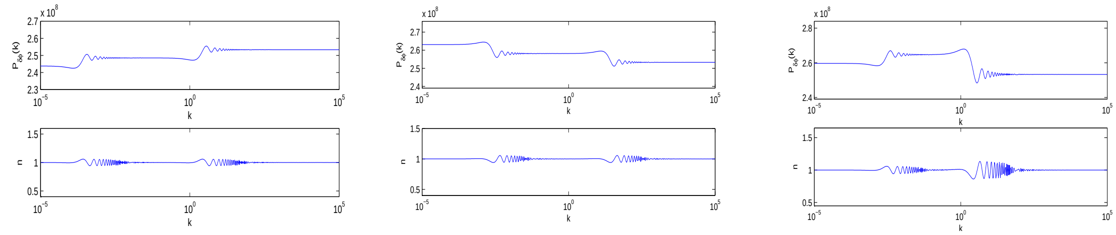
When the potential have two discontinuous points during the field rolling down, with Eq.(7) and Eq.(13), the Bogoliubov coefficients are

$$\begin{aligned} a_2 &= [1+\frac{\sigma_2}{2ix_2}(1+\frac{1}{x_2^2})]a_1+\frac{\sigma_2}{2ix_2}(-i+\frac{1}{x_2})^2 e^{2ix_2} b_1 \\ &= a_0[1+\frac{\sigma_1}{2ix_1}(1+\frac{1}{x_1^2})][1+\frac{\sigma_2}{2ix_2}(1+\frac{1}{x_2^2})]-\frac{a_0\sigma_1\sigma_2}{(2i)^2x_1x_2}(i+\frac{1}{x_1})^2e^{2ix}(-i+\frac{1}{x_2})^2e^{2i(x_2-x_1)}, \\ b_2 &= -\frac{\sigma_2}{2ix_2}(i+\frac{1}{x_2})^2e^{-2ix_2}a_1+[1-\frac{\sigma_2}{2ix_2}(1+\frac{1}{x_2^2})]b_1 \\ &= -\frac{a_0\sigma_2}{2ix_2}(i+\frac{1}{x_2})^2[1+\frac{\sigma_1}{2ix_1}(1+\frac{1}{x_1^2})]e^{-2ix_2}-\frac{a_0\sigma_1}{2ix_1}(i+\frac{1}{x_1})^2[1-\frac{\sigma_2}{2ix_2}(1+\frac{1}{x_2^2})]e^{-2ix_1}. \end{aligned} \tag{15}$$

So

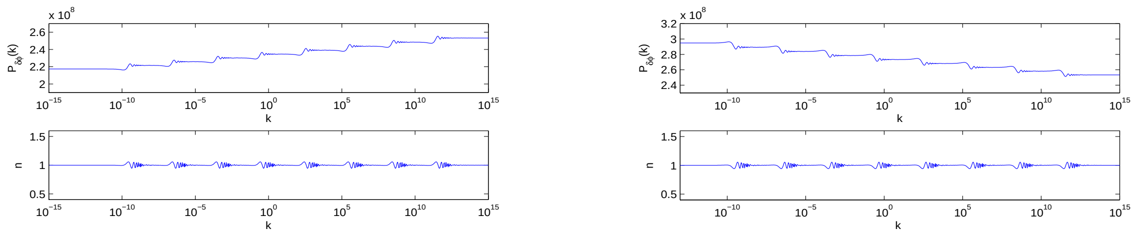
$$\begin{aligned} v \approx & \frac{a_0}{x}\{1+\frac{\sigma_1}{x_1}(i+\frac{1}{x_1})(\frac{sinx_1}{x_1}-cosx_1)e^{-ix_1}+\frac{\sigma_2}{x_2}(i+\frac{1}{x_2})(\frac{sinx_2}{x_2}-cosx_2)e^{-ix_2} \\ & +\frac{\sigma_1\sigma_2}{x_1x_2}(i+\frac{1}{x_1})e^{-ix_1}(\frac{sinx_2}{x_2}-cosx_2)[(1+\frac{1}{x_1x_2})cos(x_1-x_2)+(\frac{1}{x_2}-\frac{1}{x_2})(sin(x_1-x_2)]\}. \end{aligned} \tag{16}$$

We plot numerically $\mathcal{P}_{\delta\varphi}$ and $n_{\delta\varphi}$ in Fig.1. Fig.1 shows $\mathcal{P}_{\delta\varphi}$ has two steps and there is generally oscillations around each step. However, the oscillations only occur in a narrow region and the amplitude of oscillation rapidly decay. For $n_{\delta\varphi}$, there is also vibration in the corresponding narrow region and the amplitude is small and rapidly decay. In the Fig.1(a) and (b), the height of both steps are same because of $|\sigma_1| = |\sigma_2|$, and the power spectrum



(a) (b) (c)

FIG. 1: $\mathcal{P}_{\delta\varphi}$ *and* $n_{\delta\varphi}$ with two steps: (a) $\sigma_1 = -1/35,\ \sigma_2 = -1/35$; (b) $\sigma_1 = 1/35,\ \sigma_2 = 1/35$; (c) $\sigma_1 = -1/35,\ \sigma_2 = 1/15$. Where the unit of k is Mpc^{-1}.



(a) (b)

FIG. 2: $\mathcal{P}_{\delta\varphi}$ *and* $n_{\delta\varphi}$ with eight steps: (a) $\sigma_j = -1/35$; (b) $\sigma_j = 1/35,\ j = 1, 2, ..., 8$. Where the unit of k is Mpc^{-1}.

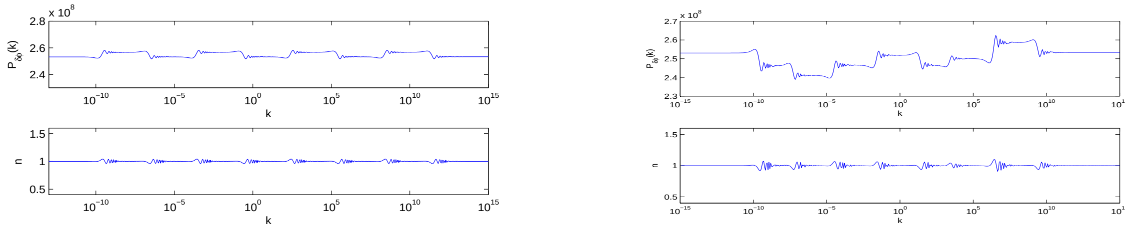
is 'up' when $\sigma < 0$ but 'down' when $\sigma > 0$. In the Fig.1(c), the power spectrum is firstly 'up' and then 'down' because of $\sigma_1 < 0,\ \sigma_2 > 0$ and the level of downward jump is larger than upward because of $|\sigma_1| < |\sigma_2|$, so the power spectrum of the large scale to little scale perturbations 'jump' down because of $\sigma_1 + \sigma_2 > 0$.

Further, although the spectrum experiences many time jumps and the value of σ is random, the values of the power spectrum on smallest and largest k may be same when

$$\sum_{j=1}^{n} \sigma_j = 0. \tag{17}$$

This can be observed as follows.

Though the simplest case in Fig.1 has grasped the points of multiple step-like behavior, it is still interesting to check the detailed case for the effective potential with $n > 2$ discontinuous points. We solve Eq.(11) numerically for the equation including eight delta functions, and the results are plotted in Figs.2 and 3. In general, the number of the 'steps' equals the number of the discontinuous points in the effective potential, and the shape of the spectrum depends on the parameter σ_i. In the Fig.2(a), the power spectrum jumps 'up'



(a) (b)

FIG. 3: $\mathcal{P}_{\delta\varphi}$ *and* $n_{\delta\varphi}$ with eight steps: (a) $\sigma_j = (-)^j 1/35,\ j = 1, 2, ..., 8$; (b) $\sigma_1 = 1/24, \sigma_2 = 1/32, \sigma_3 = -1/30, \sigma_4 = -1/32, \sigma_5 = 1/32, \sigma_6 = -1/48, \sigma_7 = -1/20, \sigma_8 = 1/32$. Where the unit of k is Mpc^{-1}.

eight times with the same value of $\sigma_i < 0$, and is alike with an upstairs. According to the value of $\sum_{j=1}^{8} \sigma_j$, the power spectrum show a large 'jump' from the largest scale to smallest scale perturbations, while it is inverse in the Fig.2(b). The Fig.3(a) shows a 'Great Wall' and Fig.3(b) shows a random wall, and the values of both the power spectrum on smallest and largest k are same with Eq.(17).

IV. DISCUSSION

We have showed that if the inflaton effective potential has multiple discontinuous points in the first derivative, the spectrum of primordial perturbation will be multiple step-like. We have given a general analysis by applying a simple model. Actually, For a nonlinear effective potential with multiple discontinuous points in the first derivative, the conclusion is the same as what we discuss in the paper, only if in the region $\varphi_j < \varphi < \varphi_{j-1}$, the potential can generate the flat power spectrum. In principle, as long as the height of step is low enough, the result of spectrum will be consistent with observations. However, it can be noticed that one or several significant steps might bring unexpected feature in CMB, which is interesting to further study.

The oscillation in primordial spectrum has appeared in many inflationary models,but a multiple step-like spectrum shows a new feature. Though there is generally oscillations around each step, the oscillations only occur in a narrow region and the amplitude of oscillation rapidly decay; the multiple step exert little influence on the whole spectrum except several unusual points, especially in Fig.3. Therefore the result given here might have inter-

esting applications in future.

Theoretically, the smooth inflation potential is perfect, but it is probable that, in reality, due to some reasons or mechanisms, the potential is rude or discontinuous in its first derivative, and the smooth only is local. On the landscape, a potential with valleys, hills, and steep as well as some shallow regions is more generic. The effective potential may be multiple discontinuous points in the first derivative, so the spectrum of primordial perturbation will be similar to the discussion in the paper.

In Ref.[15] for meandering inflation, it is argued that during inflation, inflaton would meander in a complicated multi-dimensional potential, thus during different phases the slope of potential is different, and its change is random. In this model, the effective potential is similar to that used here, thus it might be possible that one could find multiple step-like feature in its power spectrum.

In Ref.[38], the effect of the decay of fields during inflation, which leads to a staggered inflation [24], on the primordial spectrum is computed. Whenever a field decays, its associated potential energy is transferred into radiation, causing a jump in the equation of state parameter. Thus there are the discrete steps in the power spectrum. The jump in the equation of state parameter in certain sense might be equivalent to the change of the slope of effective potential, thus in this sense the result is slightly similar to that here.

Recent CMB observations seems to favor the primordial power spectrum with features. We will back to a detailed compare of a multiple step-like power spectrum with CMB observation, which will certainly impose the constrains on the number of steps and its height.

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